X_{Ao}, X_{Ab} = mole fraction of component A in water, at interface, bulk

Greek Letters

α = angle of stream bottom from horizontal, deg
γ_m = a defined concentration, $M_A/\phi_m Q$, m/l^3
Γ_v = volumetric flow rate per unit channel bottom width, l$^3/t \cdot$ l
π = 3.1416...
ρ_A = density of material A, m/l^3
σ = interfacial tension, F/l
θ, θ_A = time, lifetime of spillage, t
ν = kinematic viscosity, μ/ρ, l$^2/t$
μ = viscosity, m/l $\cdot$ t
ϕ_m = defined time, $\rho_A L/k_x X_{Ao}$, t

Subscripts

A = chemical component A
cp = constant area pool
d = drop or droplet
g = glob
vp = variable area pool
w = water
x = vertical component

LITERATURE CITED

Bird, R. B., W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena,* p. 640, Wiley, New York (1960).

Brutsaert, W., Heat and Water Vapor Exchange Between Water Surface and Atmosphere, EPA-R2-73-259, Office of Res. and Monitoring," U.S.E.P.A., Washington, D. C. (1973).

Chen, E. C., J. C. K. Overall, and C. R. Phillips, "Spreading of Crude Oil on an Ice Surface," *Can. J. Chem. Eng.,* **52**, 71 (1971).

Dahm, B. B., R. J. Pilie, and J. P. Lafornara, "Technology for Managing Spills on Land and Water," *Env. Sci. Technol.,* 8, 1076 (1974).

Daigre, G. W., Dow Chemical Company provided data on the chloroform spill and water sample analysis results, private communication (Apr. 1, 1976).

Davis, C. V., ed., *Handbook of Applied Hydrology,* p. 1204 McGraw-Hill, New York (1962).

Dilling, W. L., N. B. Tefertiller, and G. J. Kallos, "Evaporation Rates and Reactivities of Methylene Chloride, Chloroform, 1,1,1-Trichloroethane, Trichloroethylene, Tetrachloroethylene, and Other Chlorinated Compounds in Dilute Aqueous Solutions," *Env. Sci. Technol.,* **9**, No. 9, 833 (Sept. 1975).

Dzubur, I., and H. Sawistowski, "Effect of Direction of Mass Transfer on Breakup of Liquid Jets," *Proc. Intern. Solvent Extraction Conf.,* 1, 379 (1974).

Edge, R. M., and I. E. Kalafatoglu," The Breakup of Chlorobenzene Drops Falling Freely Through Water," *ibid.* (1974).

Fay, J. A., "Physical Processes in the Spread of Oil on a Water Surface," *Proc. Joint Conf. on Prevention and Control of Oil Spills,* p. 463, API, Washington, D. C. (1971)

Harbeck, G. E., Jr., "A Practical Field Technique for Measuring Reservoir Evaporation Utilizing Mass-Transfer Theory," *Geol. Survey Prof. Paper 272-E* U.S. Govt. Printing Office, Washington, D. C. (1962).

Hayworth, C. B., and R. E. Treybal, "Drop Formation in Two-Liquid-Phase Systems," *Ind. Eng. Chem.,* **42**, 1174 (1950).

Hu, S., and R. C. Kintner, "The Fall of Single Liquid Drops Through Water," *AIChE J.,* **1**, 42 (1955).

Jones, K. R., and D. Verser, "Dispersion and Dissolution of Soluble, High Density Chemicals Spilled in Aqueous Environments," Univ. Ark., Fayetteville (May, 1974).

Kappeser, R., R. Greif, and I. Cornet, "Evaporation Retardation by Monolayers," *Science,* **166**, 403 (1969).

Kramers, H., and P. J. Kreyger, "Mass Transfer between a Flat Surface and a Falling Liquid Film," *Chem. Eng. Sci.,* **6**, 42 (1956).

Langmuir, I., "Oil Lenses on Water and the Nature of Monomolecular Expanded Films," *J. Chem. Physics,* 1, 756 (1933).

Mackay, D., and R. S. Matsugu," Evaporation Rates of Liquid Hydrocarbon Spills on Land and Water," *Can. J. Chem. Eng.,* **51**, 434 (1973).

Meister, B. J., and G. F. Scheele, "Prediction of Jet Length in Immiscible Liquid Systems," *AIChE J.,* **15**, 689 (1969).

———, "Drop Formation from Cylindrical Jets in Immiscible Liquid Systems," *ibid.* (1969).

Neely, W. B., G. E. Blau, and A. Turner, Jr., "Mathematical Model Predicts Concentration-Time Profiles Resulting from Chemical Spill in a River," *Env. Sci. Technol,* **10**, 72 (Jan., 1976).

O'Brien, J. A., "Oil Spreading on Water from a Stationary Leaking Source," *Chem. Eng.,* CE 407 (Dec., 1970).

Pilie, R. S., et al., "Methods to Treat, Control and Monitor Spilled Hazardous Materials," pp. 127-136, U.S.E.P.A., Cincinnati, Ohio, *EPA-670/2-75-042* (1975).

Schooley, A. H., "Evaporation in the Laboratory and at Sea," *J. Mar. Res.,* 27, 335 (1969).

Siemer, W., and J. F. Kaufman, "Tropfenbilding in Flusskgkeiten an Dusen bei hohen Durchcatz," *Chemie-Eng. Tech.,* **29**, p. 63, 32 (1957).

Treybal, R. E., *Mass-Transfer Operations,* p. 63, McGraw-Hill, New York (1968).

Vijayan, S., and A. B. Ponter, "Coalescence in a Laboratory Continuous Mixer-Settler Unit: Contributions of Drop/Drop and Drop/Interface Coalescence Rates on the Separation Process," *Proc. Int. Solvent Extraction Conf.,* 1, 591 (1974).

Manuscript received June 14, 1976; revision received April 13, and accepted May 12, 1977.

Magnetothermal Convection of Oxygen Gas In Nonuniform Magnetic Fields

DONALD C. CLARK
and
WALLACE I. HONEYWELL

Chemical Engineering Department
University of Houston
Houston, Texas 77004

Natural convection heat transfer through oxygen gas may be significantly altered by the application of a nonuniform magnetic field. An additive contribution to the point velocity arises from the temperature dependence of the magnetic body force on the fluid. Theoretical predictions are in close agreements with experimental results.

SCOPE

In 1972, Honeywell et al. (1972) reported an unexpectedly large increase (20 to 50%) in the apparent thermal conductivity of oxygen gas at 77°K when a steady, uniform, magnetic field was applied to the gas under the influence of a small temperature gradient. These experimental results were anomalous both in the magnitude and sign of the effect. Previous experimental results and the kinetic theory of the Senftleben-Beenakker thermal conductivity effect (Bennakker and McCourt, 1970) sug-

Correspondence concerning this paper should be addressed to Wallace I. Honeywell. Donald C. Clark is with The Rohm and Haas Company, Philadelphia, Pennsylvania, 19132.

gested that only decreases in the thermal conductivity in the order of 1% maximum were possible. Small deviations from the normal S-B effects at the low pressure limit of the measurements were rationalized by Beenakker et al. (1971) on the basis of complicated field effects on the transport coefficients. But there has been no explanation of the large positive deviations observed by Honeywell at higher pressures (2.7 to 4.7 kN/m^2).

The effects of nonuniform electric fields on natural convection heat transfer have been reported previously (Senftleben, 1936; Kronig and Schwartz, 1949; Ashman and Kronig, 1950; Lykoudis and Yu, 1963; Kibler and Wiley, 1972). In the experimental results, increases in heat transfer were observed in the presence of intense electric field gradients; the theoretical treatments explained the results in terms of large electrostrictive effects in the gas induced by the intense electric fields. Analogous experimental and theoretical results in the presence of large magnetic field gradients at room temperature have also been reported. Klauer et al. (1941) and Carruthers and Wolfe (1968) reported experimental observations of such effects. The latter authors presented a simplified theoretical analysis wherein the magnetic field-field gradient product necessary to balance the buoyant force of convection was calculated. The effects were generally attributed to the changes in the magnetic body forces on the sample gas due to temperature effects on the paramagnetic susceptibility of the gas.

The special case of natural convection in an electrically conducting fluid in the presence of a uniform magnetic field has been considered theoretically (Gershuni and Zhukhovitskii, 1958). Following this treatment, Park and Honeywell (1973) developed a general theory of magnetothermal convection of gases in homogeneous magnetic fields in terms of the magnetogasdynamic momentum equations. They considered the case of perfectly uniform applied magnetic fields and applied the results to nonconducting fluids with vanishing Hartmann number; that is, the induced magnetic fields due to flow were assumed negligibly small. Under these conditions, the only nonvanishing magnetic contribution to the momentum equation is the magnetic pressure, which is superimposed on the hydrostatic pressure. The net result is a positive, magnetically induced contribution to heat transfer at all magnetic fields, temperatures, and pressures. Although the theoretical dependence of the observable variables agreed well with the experimental measurements of Vevai (1973), the magnitude of the predicted effects was too small by several orders of magnitude. The discrepancies were attributed to differences in geometry between the experimental cell and the simplified theoretical analysis.

In an attempt to determine the causes of the discrepancy between the observed and predicted effects, we decided to investigate the effect of the presence of small inhomogeneities in the applied magnetic fields, specifically, the presence of small magnetic field gradients in the otherwise homogeneous field. This paper presents the results of the application of magnetogasdynamic theory to this problem.

CONCLUSIONS AND SIGNIFICANCE

In natural convection heat transfer, an additional body force on the fluid can be induced through the application of a magnetic field gradient. This is shown by retaining the field gradient term in the momentum equation from magnetogasdynamic theory. The result is an additional contribution to the total heat transfer. The analysis is developed and applied to the special case of a nonionized, nonconducting fluid such as oxygen gas. This magnetoconvection effect is different from the magnetic pressure effects on density suggested earlier by Park and Honeywell (1973). It arises from the temperature sensitivity of the magnetic body force through the magnetic permeability of the fluid.

The theoretical development parallels that for ordinary Grashof free convection heat transfer. Applied to semiinfinite vertical flat plates, the theory predicts the superposition of a contribution to the fluid point velocity which *is dependent on the product of the magnetic field and* the field gradient. In dimensionless form, the velocity may be expressed as

$$U_z^* = \frac{1}{12}(Gr + Mg)(x^{*3} - x^*) \qquad (1)$$

where the new dimensionless group, designated the Magnetogradient convection number, is defined as $Mg \equiv h\gamma \, \Delta T D^3/\nu_o^2$, with h being the field-field gradient product. The concommitant heat transfer becomes

$$Q = \overline{\rho C_v \nu_o} \frac{(Gr + Mg)}{180} \Delta T \qquad (2)$$

The magnetic field gradient contribution can be either positive or negative, depending on the direction and geometry of the applied magnetic field.

Theoretical calculations suggest that very small magnetic field gradients can be effective in altering natural convection heat transfer through oxygen gas, especially at low temperatures. The theory also suggests an explanation for the large magnetic field gradient effects observed with air at room temperature. The strong inverse temperature dependence of the phenomenon may explain why previous experiments using nearly homogeneous magnets have not observed the effect. Comparison of the predictions from a semiinfinite flat plate model with the experimental results for a closed, concentric cylinder cell shows fair agreement over the range of available data. The parametric dependencies of the measured temperatures, pressures, and overall temperature gradients are predicted reasonably well by the theory, but a serious discrepancy between the theoretical and observed magnetic field dependence remains.

The implications of these results have possible impact on applications involving heat transfer through gaseous oxygen and air. Other paramagnetic gases such as nitric oxide can be similarly affected. The natural free convection in these gases can be significantly increased, decreased, nullified, or even reversed in direction of flow by varying the magnitude and sign of the applied magnetic field gradient. An external magnet may thus be used to control the effect of gravity on the natural convection process.

THEORY

The principal elements of the analysis of Park and Honeywell (1973) are followed here except that magnetic pressure effects are assumed negligibly small. Under the magnetogasdynamic and Boussinesq approximations, the steady state conservation equations for mass, momentum, and energy in the presence of an applied magnetic field are (in dimensional form)

$$\vec{\nabla} \cdot \vec{u} = 0 \quad (3)$$

$$\rho \vec{u} \cdot \vec{\nabla} \vec{u} = - \vec{\nabla} P_{\text{hyd}} + \eta \vec{\nabla}^2 \vec{u} + \rho \vec{g} + \mu \vec{H} \cdot \vec{\nabla} H \quad (4)$$

$$\vec{u} \cdot \vec{\nabla} T = \alpha \vec{\nabla}^2 T \quad (5)$$

We now consider the specialized application to oxygen gas confined between parallel, semiinfinite vertical flat plates separated by a distance $2D$, as shown in Figure 1. The plates are held at the steady uniform temperatures T_1 and T_2, with $T_1 - T_2 > 0$. A steady magnetic field H_x is applied along the x direction as shown. Unlike the treatment of Park and Honeywell, where this field is assumed perfectly uniform, we now allow for the practical case where the field can vary in magnitude along the y and z coordinates.

Considering only the z component of the equation of motion (4), we have the pressure gradient in the z direction as the sum of the inertial term, the gravity term, and an additional magnetic body force term due to the presence of the magnetic field gradient:

$$\frac{\partial P_{\text{hyd}}}{\partial z} = \eta \frac{\partial^2 u_z}{\partial x^2} + \rho g - \mu H_x \frac{\partial H_x}{\partial z} \quad (6)$$

It is the presence of the last term, the magnetic body force term, which was neglected by Park and Honeywell under the assumption of uniform magnetic fields. We have also neglected the contribution involving the Hartmann number, a good approximation for nonionized, nonconducting fluids such as oxygen gas.

Solution of the energy equation (5) under the above conditions results in the linear temperature distribution

$$T = T_o - \frac{1}{2} \frac{x}{D} (T_1 - T_2) \quad (7)$$

The variation in temperature with position affects the gravity and magnetic body forces through the density $\rho(T)$ and the magnetic permeability $\mu(T)$, respectively. Expanding both functions in Taylor's series about the mean temperature and retaining only the first terms, we have

$$\rho = \rho_o[1 - \beta(T - T_o)] \quad (8)$$

$$\mu = \mu_o[1 - \gamma(T - T_o)] \quad (9)$$

where β is the coefficient of thermal expansion $\beta \equiv - 1/\rho_o(\partial\rho/\partial T)_{pT_o}$, and γ is the thermal coefficient of the magnetic permeability $\gamma \equiv - 1/\mu_o(\partial\mu/\partial T)_{pT_o}$. As mentioned above, the magnetic pressure terms which accounted for the magnetoconvection contribution of Park and Honeywell are assumed negligibly small.

In the absence of x or y fluid motion, the hydrostatic pressure is a function only of the vertical z direction, and

$$\frac{\partial P_{\text{hyd}}}{\partial z} = + \rho_o g - \mu_o H_x \frac{\partial H_x}{\partial z} \quad (10)$$

Combining this equation and Equations (8) and (9) with Equation (6), we get, after rearrangement

$$\eta \frac{\partial^2 u_z}{\partial x^2} = \rho_o \beta g (T - T_o) - \mu_o \gamma H_x \frac{\partial H_x}{\partial z} (T - T_o) \quad (11)$$

The physical meaning of this equation is that in steady state the viscous forces are just balanced by the net result of the gravity and magnetic body forces.

Inserting the linear position dependence of the temperature from Equation (7), and setting the viscosity equal to the temperature averaged viscosity, we have

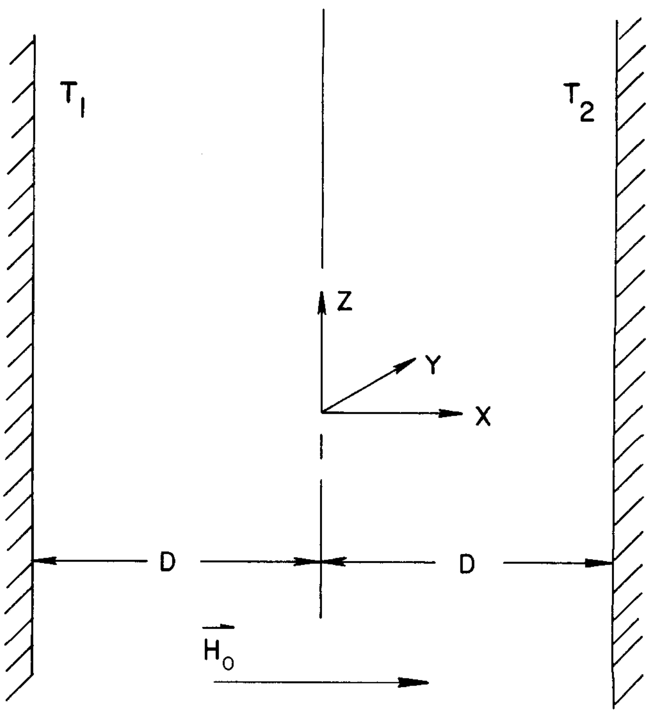

Fig. 1. Flat plate geometry showing the direction of the applied magnetic field.

$$\eta_o \frac{\partial^2 u_z}{\partial x^2} = - \frac{\Delta T}{2} \frac{x}{D} \left(\rho_o \beta g - \mu_o \gamma H_x \frac{\partial H_x}{\partial z} \right) \quad (12)$$

This is the usual simplified equation for natural thermal convection between flat plates, with the additional term accounting for the presence of the magnetic field gradients.

We now proceed to solve Equation (12) with the usual boundary conditions corresponding to zero slip at the wall and conservation of mass in a closed flow system:

B.C.

$$u_z = 0 \quad \text{at} \quad x = \pm D$$

$$\int_{-D}^{D} u_z(x) dx = 0$$

For simplification in the solution now, we restrict the value of the field-field gradient product to one which is constant h independent of position in the flow field:

$$h \equiv H_x \left(\frac{\partial H_x}{\partial z} \right)_{x,y} = \text{constant}$$

In the practical application to experiment considered below, this restriction corresponds to nearly uniform fields with a relatively small constant field gradient superimposed.

With the restrictions noted, the solution to Equation (12) results in the velocity profile

$$u_z = - \frac{1}{12} \frac{\Delta T}{D\eta_o} (\rho_o \beta g - \mu_o \gamma h)(x^3 - xD^2) \quad (13)$$

We note especially that for physical conditions where h is not a constant value, this simple form does not apply. Expressing the result in dimensionless form, we have

$$u_z^* = \frac{1}{12} [Gr + Mg](x^{*3} - x^*) \quad (14)$$

where Gr is defined as

$$Gr \equiv - g\beta \frac{\Delta T D^3}{\nu_o^2}$$

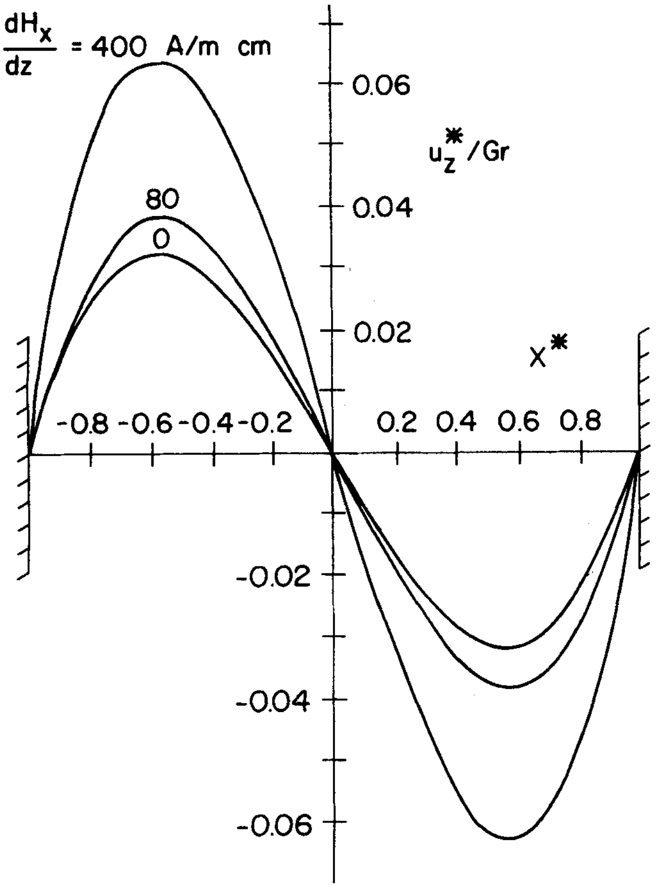

Fig. 2. Velocity profiles for oxygen gas at 77°K.

and a new dimensionless group Mg, designated the Magnetogradient convection number, is defined as

$$Mg \equiv h\gamma \frac{\Delta T D^3}{\nu_o^2} \frac{\mu_o}{\rho_o}$$

This new dimensionless group characterizes the magnetothermal contribution to the total heat transfer in constant magnetic field-field gradient conditions, just as the Grashof number characterizes the natural thermal convection contribution under the approximation of constant gravity. The presence of this new term results in a linearly superimposed contribution to the velocity in naturally convecting fluids subjected to an applied magnetic field gradient. Since the sign of the field-field gradient product can be either positive or negative, depending on the direction and geometry of the applied field, either increases or decreases in the natural convection velocity can be induced.

Theoretical velocity profiles may be expressed conveniently by normalizing the dimensionless velocity with the Grashof number

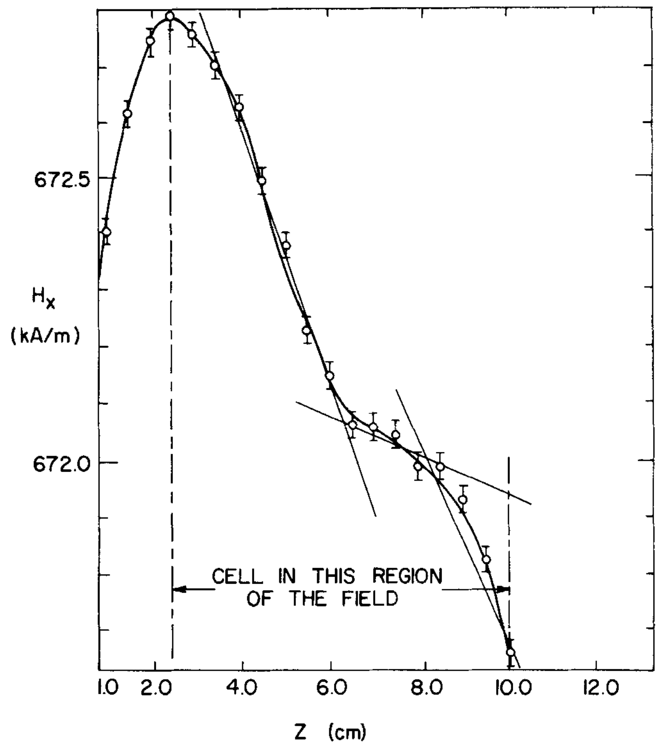

Fig. 3. Field profile of the magnet used by Vevai (1973).

$$\frac{u_z^*}{Gr} = \frac{1}{12}(1 + \Phi)(x^{*3} - x^*) \qquad (15)$$

where the parameter $\Phi \equiv Mg/Gr$ provides a convenient measure of the relative importance of the magnetically to the gravitationally induced contribution to the velocity. The importance of even very small field gradients in large applied magnetic fields is illustrated in Figure 2, where the normalized dimensionless velocity profiles of Equation (15) are given for field gradients of 0, 80, and 400 A/m cm in an applied field of 1.2 MA/m. These results are for the case of oxygen at T_o = 77°K and for a Curie constant C of 1.02 m³K/mole. These results show clearly that relatively small magnetic field gradients can cause substantial increases in the fluid velocity. The high sensitivity indicates that further increases in the magnitude of the magnetic field gradient will cause the flow to be controlled entirely by the magnetic body forces. Although these results apply only for laminar flow, they may explain the marked effects extending into the turbulent flow region which were observed by Carruthers and Wolfe (1968).

The contributions to the total heat transfer Q may be estimated theoretically from the equation (following the treatment of Gershuni and Zhukhovitskii, 1958)

$$Q = \bar{\rho}\bar{C}_v \int_{-D}^{0} u_z(x)(T - T_o)dx \qquad (16)$$

TABLE 1. MAGNETOCONVECTIVE HEAT TRANSFER IN MILLIWATTS AT VARIOUS CONDITIONS FOR THE EXPERIMENTAL DATA OF VEVAI (1973).

Field strength (MA/m)	$p = 2.56$, $T_o = 89.5$, $\Delta T = 22.7$		$p = 4.72$, $T_o = 112.0$, $\Delta T = 22.6$		$p = 4.75$, $T_o = 81.7$, $\Delta T = 12.5$		$p = 4.75$, $T_o = 98.0$, $\Delta T = 22.6$		$p = 3.73$, $T_o = 96.2$, $\Delta T = 22.6$		$p = 4.77$, $T_o = 86.3$, $\Delta T = 22.6$	
	Expt.*	Theory	Expt.*	Theory	Expt.*	Theory	Expt.*	Theory	Expt.*	Theory	Expt.*	Theory
0.561	0.116	0.318	0.106	0.237	0.142	0.361	0.210	0.470	0.225	0.558	0.259	0.906
0.702	0.229	0.275	0.192	0.298	0.278	0.455	0.390	0.592	0.479	0.703	0.611	1.142
0.844	0.398	0.331	0.316	0.358	0.519	0.546	0.727	0.711	0.844	0.844	1.072	1.372
0.979	0.628	0.384	0.488	0.416	0.770	0.634	1.080	0.826	1.294	0.980	1.647	1.952
1.12	0.951	0.440	0.732	0.477	1.113	0.727	1.572	0.946	1.954	1.123	2.351	1.826

* Vevai (1973).

per unit width of the system in the y direction, where C_v is the temperature averaged heat capacity at constant volume over the region 0 to $-D$. Inserting Equation (14) and integrating, we have

$$Q = \overline{\rho C_v}\, \nu_o \frac{(Gr + Mg)}{180} \Delta T \qquad (17)$$

The net contribution to the convection heat transfer induced by the magnetic field gradient is then

$$Q_{Mg} = \overline{\rho C_v}\, \nu_o Mg \frac{\Delta T}{180} \qquad (18)$$

Examining only those measurable variables which are sensitive to changes in pressure, temperature, and magnetic field, we have

$$Q_{Mg} \alpha \frac{p^2 \Delta T^2}{\eta_o T_o^4} H_x \frac{dH_x}{dz} \qquad (19)$$

The perfect gas approximation for the density has been used here. This form of the result is convenient for experimental verification of the theory, as described below.

COMPARISON WITH EXPERIMENT

The extensive measurements of Vevai (1973) provide experimental tests of Equations (18) and (19). The data were obtained in a concentric cylinder cell 3.05 mm ID 31.8 mm OD at several conditions of pressure, temperature, field strength, and overall temperature gradient. Parametric studies were made over the ranges $0.557 < H_x < 1.122$ MA/m, $0.67 < p < 4.8$ kN/m^2, $12 < \Delta T < 22$°C, and $79 < T_o < 112$°K. With the exception of ΔT, the experimental parameters were varied independently to obtain absolute magnitudes as well as functional dependencies of the results. Since ΔT and T_o could not be varied independently of each other, only the functional dependence of ΔT could be obtained.

The theoretical analysis of Park and Honeywell (1973) assumed that the applied magnetic field used for the measurements was perfectly uniform. In subsequent investigations, Clark (1975) discovered the presence of small nonuniformities in the magnetic fields which had been used to obtain the data. Precise measurements of the field were made along the z axis to a sensitivity of one part in 10^5 in relative field strength. Time stability measurements were also made to ascertain that the measured fields were free of drift. A representative set of field strength data is shown in Figure 3. At each of the additional sets of data (0.67, 0.76, 0.88, 0.95, 1.03, and 1.11 MA/m), regions of relatively high linearity of the field with distance were present. To account for the variations, however, effective field gradients were estimated by geometrically averaging the approximately linear regions indicated. The effective field gradient for each data set was -160 A/m cm within experimental error. The field direction was determined to be negative relative to the coordinates of Figure 3.

With this value of the effective magnetic field gradient, calculations were made using Equation (18) for conditions corresponding to the experimental data. The log mean sample thickness was used in the calculations to account for the difference in geometry between the theoretical flat plate model and the actual cylindrical cell geometry of the experiments. The results are given in Table 1 for various magnetic fields. The theoretical predictions agree reasonably well in magnitude over the range of experimental results. This agreement is a significant improvement over the results of Park and Honeywell (1973).

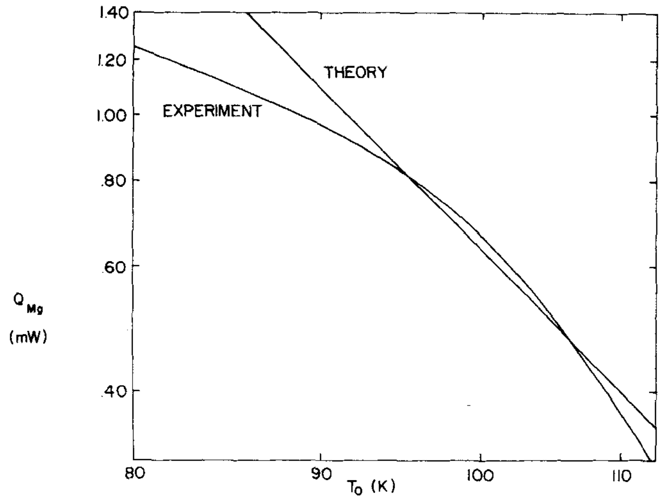

Fig. 4. Temperature variation of the magnetoconvection heat transfer of oxygen gas at 0.844 MA/m field strength, −160 A/m cm field gradient, 4.8 kN/m^2 pressure, and ΔT of 22.6°C.

The theoretically predicted temperature dependence of the heat flux is found to be $T_o^{-5.07}$ from Equation (19) when incorporating a least-squares approximation of an empirical correlation for the low temperature behavior of the oxygen gas viscosity. The theoretical prediction is illustrated in Figure 4 along with the smoothed experimental curve. The experimental dependency ranges between $T_o^{-4.8}$ and $T_o^{-5.2}$.

According to Equation (19), the heat flux should vary quadratically with the pressure. The experimental results show an approximate cubic dependence below 2.7 kN/m^2, changing smoothly to an approximate quadratic dependence at pressures between 3.3 and 4.8 kN/m^2. The quadratic dependence of ΔT predicted theoretically is in agreement with the indirect experimental determination of this dependency obtained by Vevai (the agreement is reportedly best for $\Delta T < 10$°C).

The most difficult theoretical result to reconcile with experiment is the magnetic field dependence of the magnetoconvective heat flux. According to Vevai (1973), the experimental evidence clearly indicates a cubic field dependence, whereas the theory is linear in the field-field gradient product. With the constant effective field gradient for all the data, this discrepancy cannot be explained at this time.

ACKNOWLEDGMENT

The authors wish to acknowledge financial support from the National Science Foundation (grant GK 23009) for this work.

NOTATION

- C = Curie constant for paramagnetically susceptible matter, m^3°K/mole
- C_v = heat capacity at constant volume, J/g°K
- D = characteristic distance, m
- $\vec{H}$ = externally applied magnetic field, A/m
- g = acceleration due to gravity at the surface of the earth, 9.807 m/s^2
- Mg = dimensionless magnetogradient convection number, $\gamma D^3 \Delta T/\nu_o^2\ \mu_o/\rho_o\ h$
- Gr = dimensionless Grashof Number, $-\beta D^3 g \Delta T/\nu_o^2$
- h = constant field-field gradient product, $H_x\ \partial H_x/\partial z$, A^2/m^2 cm
- H_x = externally applied constant magnetic field in the x direction, A/m
- M = molecular weight, g/mole

Mc = dimensionless magnetoconvection number after Park and Honeywell
p_{hyd} = hydrostatic pressure, N/m^2
Q = total heat transfer rate, J/s
Q_{Mg} = magnetothermal contribution to the total heat transferred due to the magnetic field gradient effect, J/s
T = temperature, °K
T_o = average T over ΔT, °K
T_1 = hot plate temperature, °K
T^* = dimensionless temperature, $T/\Delta T$
T_2 = cold plate temperature, °K
ΔT = temperature difference across the fluid, $T_1 - T_2$, °C
$\vec{u}$ = fluid velocity vector, m/s
$\vec{u}^*$ = dimensionless velocity vector, $\vec{u}D/\nu$
x^* = dimensionless coordinate, x/D

Greek Letters

α = thermal diffusivity, $\Lambda_o/\rho C_v$, m^2/s
β = coefficient of thermal expansion, $= -1/\rho\ (\partial\rho/\partial T)_p|_{T_o}$, K^{-1}
η = dynamic viscosity, Ns/m^2
η_o = average η over ΔT, Ns/m^2
Λ_o = standard thermal conductivity, W/m °K
μ = magnetic permeability
μ_o = average μ over ΔT
ν_o = average kinematic viscosity over ΔT, m^2/s
ρ = density, g/m^3
ρ_o = average ρ over ΔT, g/m^3
$\vec{\nabla}$ = del operator, m^{-1}
γ = thermal coefficient of magnetic permeability, $= -1/\mu_o\ (\partial\mu/\partial T)_p|_{T_o}$, $°K^{-1}$
Φ = parameter Mg/Gr

LITERATURE CITED

Ashmann, G., and R. Kronig, "The Influence of Electric Fields on the Convective Heat Transfer in Liquids," *Appl. Sci. Res.*, A2, 235 (1950).

Beenakker, J. J. M., and F. R. McCourt, "Magnetic and Electric Effects on Transport Properties," *Ann. Rev. Phys. Chem.*, 21, 47 (1970).

Beenakker, J. J. M., J. A. R. Coope, and R. F. Snider, "Influence of a Magnetic Field on the Transport Coefficients of Oxygen Gas: Anomalies Associated with the $\sigma=0$ Multiplets," *Phys. Rev.*, 4A, 788 (1971).

Carruthers, J. R., and R. Wolfe, "Magnetothermal Convection in Insulating Paramagnetic Fluids," *J. App. Phys.*, 39, 5718 (1968).

Clark, D. C., M.S. thesis, Univ. Houston, Tex. (1975).

Gershuni, G. Z. and E. M. Zhukhovitskii, "Stationary Convective Flow of an Electrically Conducting Liquid Between Parallel Plates in a Magnetic Field," *Sov. Phys., JETP*, 34, 461 (1958).

Honeywell, W. I., D. G. Elliot, and J. E. Vevai, "Large Magnetic Field Effects on Oxygen Gas Thermal Conductivity at 77K," *Phys. Lett.*, 38A, 265 (1972).

Kibler, K. G., and R. Wiley, "Electrostatic Cooling," *Ind. Res.*, 50 (Apr., 1972).

Klauer, F., E. Turowsky, and T. V. Wolfe, "Untersuchungen ueber das Verhalten paramagnetisher Gase im inhomogenen Magnetfeld," *Z. Tech. Phys.*, 22, 223 (1941).

Kronig, R., and N. Schwarz, "On the Theory of Heat Transfer from a Wire in an Electric Field," *Appl. Sci. Res.*, A1, 35 (1949).

Lykoudis, P. S., and C. P. Yu, "The Influence of Electrostrictive Forces in Natural Thermal Convection," *Intern. J. Heat Mass Transfer*, 6, 853 (1963).

Park, W.-H., and W. I. Honeywell, "Magnetothermal Convection of Polyatomic Gases in Homogeneous Magnetic Fields, Part I: Theory," *Chem. Eng. Commun.*, 1, 167 (1973).

Senftleben, H., and W. Braun, "Der Einfluss Elektrisher Felder auf den Waermestrom in Gases," *Z Phys.*, 102, 480 (1936).

Vevai, J. E., Ph.D. dissertation, Univ. Houston, Tex. (1973).

Manuscript received January 11, 1977; revision received May 9, and accepted May 12, 1977.

Discrete Flow Modeling: A General Discrete Time Compartmental Model

FEG-WEN CHANG
and
THOMAS J. FITZGERALD
Department of Chemical Engineering
Oregon State University
Corvallis, Oregon 97331

A general flow model is developed which is discrete both in time and space. The stirred-tank network model (continuous time compartmental model) is summarized and compared with the discrete flow model. Efficient methods for model fitting are given and demonstrated with numerical examples.

SCOPE

A new model is presented for treating data collected from multiple probes inside a stationary flow system. It is assumed that there are sufficient probes so that the state of the system is well described by the set of concentrations measured by the probes. Matrix methods are presented to relate this new model which is discrete in both time and space to the continuous time compartmental models (stirred-tank networks) commonly used in physiological modeling. The new model and the older compartmental model are both characterized by matrices of fractional input flow coefficients which give the fraction of flow coming into one region which originated in another. It is shown that the volumes of the regions can be calculated from this fractional input matrix. The fractional input matrix can also be used to find the coefficients of the linear difference equation relating the concentration in any region to its past values.